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A THEOREM IN CONTINUED FRACTIONS.

BY DERRICK N. LEHMER.

IN an earlier note on Continued Fractions* it was proved in effect that

$$A_{n-2} = B_{n-1}$$

where A_m, B_m denote respectively the numerator and denominator of the mth convergent of the simple periodic continued fraction

$$x = q_1 + \frac{1}{q_2 +}\,\frac{1}{q_3 +} \cdots\cdot + \frac{1}{q_n +}\,\frac{1}{q_1 +} \cdots\cdot$$

occurring in the development of a quadratic surd into a continued fraction

$$\sqrt{R} = q_0 + \frac{1}{x}\cdot$$

By the aid of this relation and the theorem: Every simple periodic continued fraction is a root of a quadratic equation with integral coefficients whose roots are of contrary signs, one being greater and the other less than unity in absolute value, and the reciprocal of the negative root is equal in absolute value to the continued fraction which has the same quotients in inverse order (C. Smith, *Algebra*, p. 462),—we may show readily that the quotients in the expansion of a quadratic surd, the nth quotient being excluded, read backward and forward the same.

Consider the simple periodic continued fraction

$$y = q_1 + \frac{1}{q_2 +}\,\frac{1}{q_3 +} \cdots\cdot + \frac{1}{q_{n-1} +}\,\frac{1}{q_1 +} \cdots\cdot \tag{1}$$

obtained by omitting q_n in the value of x given above. Up to and including A_{n-1}/B_{n-1} the convergents of y are identical with those of x, so that

$$B_{n-1} = A_{n-2}\,.$$

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* ANNALS OF MATHEMATICS, ser. 1, vol. 11, p. 64.

Now the equation for y is (C. Smith, *l. c.*)

$$B_{n-1}\,y^2 + (B_{n-2} - A_{n-1})\,y - A_{n-2} = 0$$

and if $-y'$ is its negative root

$$\frac{1}{y'} = q_{n-1} + \frac{1}{q_{n-2} +}\,\frac{1}{q_{n-3} +} \cdots + \frac{1}{q_1 +}\,\frac{1}{q_{n-1} +} \cdots . \qquad (2)$$

Since $$B_{n-1} = A_{n-2}, \quad -yy' = -1 \quad \text{and} \quad \frac{1}{y'} = y.$$

Hence the continued fractions in (1) and (2) are equal, and corresponding quotients q_i and q_{n-i} are identical (Chrystal, *Algebra*, vol. 2, p. 398). Therefore the quotients of x exclusive of q_n read backward and forward the same.

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